INVESTIGATION OF WAVE FIELDS IN A FLUID LAYER AND ELASTIC HALF-SPACE EXCITED BY A NEAR-BOTTOM SOURCE OF VIBRATIONS

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Questions of a study of acoustic and seismic wave propagation in the ocean and the underlying medium, associated with problems of marine hydrolocation (sonar) and the investigation of tsunami wave predecessors caused by underwater volcanic eruptions are of great interest at this time. The simplest model, a point source of vibrations located in a fluid layer at a certain range from the bottom, is used to describe the mentioned wave processes. Wave fields in a fluid and elastic base are investigated in this paper, analytical formulas are obtained, and results of a numerical analysis are presented.

Harmonic axisymmetric vibrations caused by the vibration of a point source in a layer ($0 \leqslant R < \infty$, $0 \leqslant \Theta < 2\pi$, $0 \leqslant Z \leqslant H$) of an ideal compressible weightless fluid are studied. The layer is on an elastic half-space ($0 \leqslant R < \infty$, $0 \leqslant \Theta < 2\pi$, $-\infty < Z \leqslant 0$). There is a lumped source of harmonic vibrations of δ-function type at the point $(0, 0, h_0)$, $0 \leqslant h_0 < H$.

The velocity potential $\Phi(R, Z, t)$ of the fluid particles satisfies the wave equation [1] with right side of the form $C_0R^{-1}\delta(R)\delta(Z - h_0)e^{-i\omega t}$.

The components of the displacement vector $U_r(R, Z, t)$, $U_z(R, Z, t)$ in an elastic medium under axial symmetry satisfy the system of Lamé dynamic equations in a cylindrical coordinate system [2]. The fluid surface is free of stresses and the normal velocities are given equal on the interfacial surface between the fluid and the elastic medium. Moreover, the wave radiation conditions should be satisfied at infinity. Using the method of integral transforms, we find the potential of the fluid particle velocity and the displacement in the elastic medium:

$$\Phi(r, z, t) = \frac{C_0}{H} e^{-i\omega t} \int_\Gamma \varphi(\lambda, z) J_0(\lambda r) \lambda d\lambda,$$

$$U_r(r, z, t) = \frac{iC_0 m}{\omega H^2} e^{-i\omega t} \int_\Gamma u(\lambda, z) J_1(\lambda r) \lambda d\lambda, \quad U_z(r, z, t) = \frac{iC_0 m}{\omega H^2} e^{-i\omega t} \int_\Gamma w(\lambda, z) J_0(\lambda r) \lambda d\lambda,$$

$$\varphi(\lambda, z) = \begin{cases} \dfrac{\operatorname{sh}(\gamma(z-1))\left[m\delta_1\varkappa_2^2 s \operatorname{sh}(\gamma h) - \gamma \operatorname{ch}(\gamma h)\Delta(\lambda)\right]}{\gamma D(\lambda)}, & z > h, \\ \dfrac{\operatorname{sh}(\gamma(h-1))\left[m\delta_1\varkappa_2^2 s \operatorname{sh}(\gamma z) - \gamma \operatorname{ch}(\gamma z)\Delta(\lambda)\right]}{\gamma D(\lambda)}, & z \leqslant h, \end{cases} \tag{1}$$

$$u(\lambda, z) = \frac{s \operatorname{sh}(\gamma(1-h))\left[-\lambda(2\lambda^2 - \varkappa_2^2)e^{\delta_1 z} + 2\lambda\delta_1\delta_2 e^{\delta_2 z}\right]}{D(\lambda)},$$

$$w(\lambda, z) = \frac{s \operatorname{sh}(\gamma(1-h))\left[\delta_1(2\lambda^2 - \varkappa_2^2)e^{\delta_1 z} + 2\lambda^2\delta_1 e^{\delta_2 z}\right]}{D(\lambda)},$$

$$D(\lambda) = ms\varkappa_2^2\delta_1 \operatorname{sh}(\gamma) - \gamma \operatorname{ch}(\gamma)\Delta(\lambda), \quad \Delta(\lambda) = 4\lambda^2\delta_1\delta_2 - (2\lambda^2 - \varkappa_2^2)^2,$$

where

$$r = R/H; \; z = Z/H; \; h = h_0/H;$$

$$\varkappa_1^2 = \frac{H^2\omega^2}{V_p^2}; \quad \varkappa_2^2 = \frac{H^2\omega^2}{V_s^2} = \frac{\rho H^2\omega^2}{\mu}; \quad k^2 = \frac{H^2\omega^2}{V_0^2}; \quad s = \frac{H\omega^2}{g}; \tag{2}$$

$$m = \rho_0 gH/\mu; \quad \delta_i = \sqrt{\lambda^2 - \varkappa_i^2} \quad (i = 1, 2); \quad \gamma = \sqrt{\lambda^2 - k^2};$$

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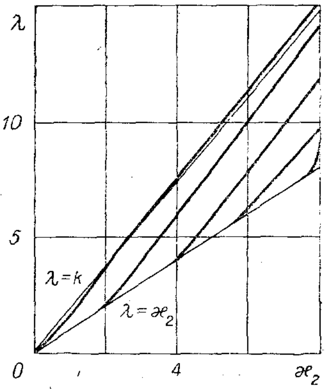

Fig. 1

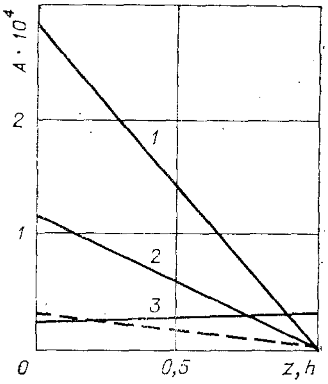

Fig. 2

μ, ρ, V_S, V_p are, respectively, the shear modulus, density, transverse and longitudinal wave velocities in the elastic half-space, ρ_0 is the fluid density, $\omega = 2\pi f$, f is the vibrational frequency of the perturbation source, V_0 is the sound speed in the fluid, C_0 is the amplitude function of the pressure, and $\delta(x)$ is the delta function. The semiinfinite contour of integration Γ in (1) coincides with the whole real line from 0 to $+\infty$, except for singularities of the integrand (zeros and branch points of $D(\lambda)$) which it bypasses from below. Such a selection of the contour is governed by the wave radiation conditions and is described in detail in [3].

Taking account of the relation between the potential and the fluid particle velocities, the velocity field in the whole space occupied by the fluid as well as the displacement field in the whole elastic half-space can be computed by using (1). The particular case for a source on the bottom of a reservoir is considered in [4].

The zeros of the denominator $D(\lambda)$ of the integrands in (1) are investigated numerically on a digital computer as a function of the reduced frequency $\varkappa_2$ (2). It is established that all the real zeros of the denominator $D(\lambda)$ lie in a certain angle formed by the lines $\lambda = \varkappa_2$ and $\lambda = k$. Only the very first zero, starting from a certain time, will emerge upward from this angle and later pass somewhat above the line $\lambda = k$. The characteristic dependence of the zeros on the frequency $\varkappa_2$ is presented in Fig. 1.

As is seen from (1) in the far zone, i.e., for $r \gg 1$, the integrands are rapidly oscillating functions because of the Bessel functions. Consequently, here numerical methods of calculating the integrals are slightly effective. Such integrals are evaluated by using residue theory. To do this, by using relationships between the Bessel functions of the first and third kind [5], the contour Γ is expanded on the whole real axis and closed in the plane where the integrand decreases. The negative real singularities of the integrand are bypassed from above. Consequently, the initial integral is represented as the sum of three components. The first is the sum of residues at the real positive poles, finite in number. A Rayleigh wave that decreases at infinity (for $r \to \infty$) or $O(r^{-1/2})$ corresponds to each term of this sum. The second component is the residues at the complex poles; it induces an exponentially decreasing contribution of order $O(e^{-\varepsilon r})$ into the total sum, where ε is the lower bound of the imaginary component of the complex poles. This component governs the penetrating wave. The third component is the integrals over the slit edges drawn from the branch points $\lambda = \varkappa_1$ and $\lambda = \varkappa_2$. The contribution of this component to the total sum for large r is of the order of $O(r^{-3/2})$. Because of the estimates presented, only the first component corresponding to the Rayleigh waves yields the main contribution to the value of the integral (1) for $|z| \ll 1$ at a sufficient distance from the source ($r \gg 1$).

The numerical analysis performed for the wave fields in the elastic and liquid media is illustrated in Figs. 2-4. The amplitude distribution functions of the velocity potential $\Phi \cdot 10^{-1}$ and the velocities V_r, V_z themselves over the depth of the fluid layer are shown by lines 1-3 in Fig. 2. The distance along the horizontals between the vibration source and the point being investigated equals 100 layer thicknesses. The source itself is at a distance h = 0.1 from the bottom. The coefficient C_0 in (1) is taken equal to 1, and the depth of the reservoir is H = 100 m. The amplitudes are computed in meters. Computations are executed for $\varkappa_2 = 2.35$. The dashed line shows the value of the amplitude function V_z at a point of distance z = 0.1 away from the bottom, and at a distance r = 100 along the horizontal from the source itself at depths h ($0 \leqslant h \leqslant 1$) in the layer. In this case the amplitude functions for Φ and V_r agree with the values of Φ and V_r in the preceding case.

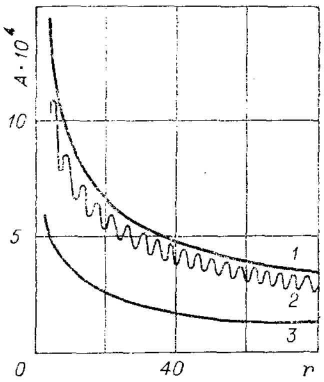

Fig. 3

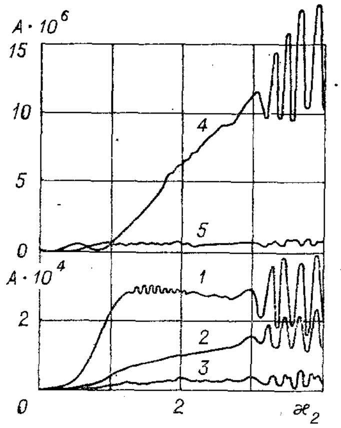

Fig. 4

The amplitude distribution functions of $\Phi \cdot 10^{-1}$, $V_z \cdot 10$, and V_r are shown in Fig. 3 by the lines 1-3, respectively, as functions of the distance r between the source and the point of recording the vibrations. As before, the source is here at a distance h = 0.1 from the bottom of the reservoir and the receiver is at the bottom itself. Computations are performed for $\varkappa_2 = 2.35$. It is clear from Figs. 2 and 3 how the wave field is distributed over the whole domain occupied by the fluid.

The dependence of the amplitude functions of $\Phi \cdot 10^{-1}$, V_r, and V_z on the bottom of the reservoir and the amplitude functions of the displacement U_r and U_z (the lines 1-5, respectively) at the surface of the elastic half-space on the reduced frequency $\varkappa_2$ is represented in Fig. 4 for the same values h = 0.1 and r = 100. Taking account of the relation (2) between the reduced frequency $\varkappa_2$ and the parameters ω, H, μ, Fig. 4 yields a representation of the dependence of the velocity potential Φ, the velocities V_r, V_z, and the displacements U_r, U_z on ω, H, μ. Analogous curves are obtained for a large number of different points of the fluid layer and the elastic half-space. They have a form similar to Figs. 2-4.

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